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MINIMUM ΔV, TWO- AND THREE-IMPULSE NON-COPLANAR ABORT MANEUVERS ONTO AN ESCAPE ASYMPTOTIC VELOCITY VECTOR FOLLOWING PREMATURE SPS SHUTDOWN DURING LOI PHASE

Mathematical Physics Branch
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MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

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PROJECT APOLLO

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Mathematical Physics Branch

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MISSION PLANNING AND ANALYSIS DIVISION
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MINIMUM-ΔV, TWO- AND THREE-IMPULSE NON-COPLANAR

ABORT MANEUVERS ONTO AN ESCAPE ASYMPTOTIC VELOCITY

VECTOR FOLLOWING PREMATURE SPS SHUTDOWN DURING LOI PHASE

By Wayne O. Laszlo

SUMMARY

The purpose of this study was to determine a class of minimum-ΔV, two-impulse and three-impulse abort trajectories (maneuvers) onto an optimal lunar escape hyperbola partially specified by a fixed asymptotic velocity vector and a minimum pericynthion. The term lunar escape means escape from the moon's sphere of influence.

Minimum-ΔV, two-impulse and three-impulse orbit transfer programs based on the accelerated gradient method were used. The programs were of the form impulse-coast-impulse (I-C-I) and impulse-coast-impulse-coast-impulse (I-C-I-C-I), respectively. Using these programs it was concluded that the best solutions are the degenerate two-impulse maneuver (C-I), $T1 = T1'' = T1_0$ and the degenerate three-impulse maneuver (C-I-C-I), $T1 = T1' = T1_0$ which require a total ΔV of 1400.074 fps and 1397.173 fps, respectively. A degenerate maneuver is defined as a maneuver in which the first impulse velocity vector = $\vec{0}$.

In another study, a typical two-impulse solution (maneuver) was determined which required a total ΔV of 2252.26 fps. This solution was considered in this document as the reference trajectory for the class of minimum-ΔV, two-impulse, and three-impulse abort trajectories generated by the accelerated gradient programs, and, in particular, for the best solutions (trajectories) within this class.

The best solutions obtained by the two-impulse and three-impulse orbit transfer programs based on the accelerated gradient method led to the following percentage decreases in total ΔV (the sum of the magnitudes of the impulses) from the reference trajectory.

For the degenerate two-impulse, $T1 = T1'' = T1_0$-hour maneuver there was a 37.8 percent decrease in total ΔV, and for the degenerate three-impulse $T1 = T1' = T1_0$-hour maneuver there was a 38.0 percent decrease in total ΔV.

INTRODUCTION

If there is no shutdown during the LOI phase, the spacecraft will be inserted into an 80-n. mi. circular orbit around the moon. However, this study was needed in the event that a premature SPS shutdown during the LOI phase forced a two-impulse or three-impulse abort maneuver from a non-nominal, elliptical, lunar parking orbit.

Reference 1 documented abort procedures in the event of premature shutdowns at various times (65, 130, 136, 148, 160, and 270 seconds) during the required 380-second SPS burn in the LOI phase. The document described a two-impulse abort procedure used when the delay time from premature shutdown to the first abort maneuver was greater than from 1 to 1.5 hours. One typical two-impulse solution, assuming a 2-hour delay from SPS shutdown to the first abort maneuver, required a total ΔV of 2252.26 fps. This solution was considered in this study as the reference trajectory (fig. 1) for the class of minimum-ΔV, two-impulse and three-impulse abort trajectories generated by the programs based on the accelerated gradient method.

This study was performed at the request of Charles E. Foggatt, Flight Analysis Branch, Mission Planning and Analysis Division.

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SYMBOLS

a, a_h	semimajor axis of the terminal escape hyperbola
$\bar{a}$, $\bar{a}_h$	semimajor axis of the optimal terminal escape hyperbola
$\bar{a}_0$	semimajor axis of the elliptical lunar parking orbit
a_1, a_2	semimajor axis of the elliptic first and second transfer conics, respectively
D_∞	declination angle of $\vec{V}_\infty$
$\sin D_\infty$	sine of D_∞

$\overline{\sin D_\infty}$	sine of D_∞ of $\vec{V}_\infty$ associated with the optimal terminal escape hyperbola
e, e_h	eccentricity of the terminal escape hyperbola
$\bar{e}$	minimum allowable eccentricity of the optimal terminal escape hyperbola
$\bar{e}_0$	eccentricity of the elliptical lunar parking orbit
e_1, e_2	eccentricity of the elliptic first and second transfer conics, respectively
F	$\sum_{i=1}^{n} \lvert\overrightarrow{\Delta V}\rvert_i$, the sum of the magnitudes of the impulses
g_1, g_2, g_3, g_4, g_5	numerical value of the first, second, third, fourth, and fifth constraints, respectively, at the converged limit attained by the two-impulse and three-impulse accelerated gradient programs. The five constraints are as follows:

first constraint . . . . $a_1 - \overline{a_1} = 0$

second constraint. . $\sin D_\infty - \overline{\sin D_\infty} = 0$

third constraint . . . . $a - \bar{a} = 0$

fourth constraint . . . . . $RA - \overline{RA} = 0$

fifth constraint . . . . $\overline{r_p} - r_p \leq 0$

LOI	lunar orbit insertion
$(\overline{r_a})_0$	apocynthion of the elliptical parking orbit
$(r_a)_1$, $(r_a)_2$	apocynthion of the elliptic first and second transfer conics, respectively
r_p	pericynthion of the terminal escape hyperbola

$\bar{r}_p$, $(\bar{r}_p)_h$	minimum allowable pericynthion of the optimal terminal escape hyperbola
$(\bar{r}_p)_0$	pericynthion of the elliptical lunar parking orbit
$(r_p)_1$, $(r_p)_2$	pericynthion of the elliptic first and second transfer conics, respectively
RA	right ascension angle of $\vec{V}_\infty$
$\overline{RA}$	right ascension angle of $\vec{V}_\infty$ associated with the optimal terminal escape hyperbola
$SV_1(\vec{R}_1, \vec{V}_1^-)$	fixed initial state vector or state vector immediately prior to the first impulse. The position components are x_1, y_1, z_1; and the velocity components are u_1^-, v_1^-, w_1^- (or $\dot{x}_1^-$, $\dot{y}_1^-$, $\dot{z}_1^-$). Components are in the X, Y, Z coordinate system
$SV_2(\vec{R}_2, \vec{V}_2^-)$	state vector immediately prior to the second impulse. The position components are x_2, y_2, z_2; and the velocity components are u_2^-, v_2^-, w_2^- (or $\dot{x}_2^-$, $\dot{y}_2^-$, $\dot{z}_2^-$). Components are in the X, Y, Z coordinate system
$SV_3(\vec{R}_3, \vec{V}_3^-)$	state vector immediately prior to the third impulse. The position components are x_3, y_3, z_3; and the velocity components are u_3^-, v_3^-, w_3^- (or $\dot{x}_3^-$, $\dot{y}_3^-$, $\dot{z}_3^-$). Components are in the X, Y, Z coordinate system
$SV_h(\vec{R}_s, \vec{V}_s)$	selenocentric state vector at the moon's sphere of influence of a preassigned terminal escape hyperbola. Components are in the X, Y, Z coordinate system
SPS	service propulsion system
t_1	total elapsed time of flight at the point of the first impulse, sec ($t_1 = 0$ at first impulse)

t_2, t_3	total elapsed time from the first impulse at the point of the second impulse and third impulse, respectively, sec
Tl_0	fixed period of the elliptical lunar parking orbit
Tl	period of the elliptic first transfer (after the first impulse) conic. It was a variable constant (parameter) for the class of two-impulse or three-impulse trajectories.
Tl', Tl"	finite value of Tl obtained when the three-impulse and two-impulse programs, respectively, were run with the period (first constraint) removed. It is the exact value of Tl, for the respective programs, which gives the smallest possible numerical value of F.
$\vec{V}_\infty$	velocity vector at infinity associated with the terminal escape hyperbola. The components in the X, Y, Z coordinate system are $(V_\infty)_x$, $(V_\infty)_y$, $(V_\infty)_z$
V_∞, $\lvert\vec{V}_\infty\rvert$	magnitude of $\vec{V}_\infty$ associated with the terminal escape hyperbola
$\bar{V}_\infty$	magnitude of $\vec{V}_\infty$ associated with the optimal terminal escape hyperbola
$\overrightarrow{\Delta V_1}$	first impulse vector whose components are Δu_1, Δv_1, Δw_1
$\lvert\overrightarrow{\Delta V_1}\rvert$, $\lvert\overrightarrow{\Delta V}\rvert_1$	magnitude of first impulse vector
$\Delta\vec{V}_2$	second impulse vector whose components are Δu_2, Δv_2, Δw_2
$\lvert\overrightarrow{\Delta V_2}\rvert$, $\lvert\overrightarrow{\Delta V}\rvert_2$	magnitude of the second impulse vector
$\overrightarrow{\Delta V}_3$	third impulse vector whose components are Δu_3, Δv_3, Δw_3
$\lvert\overrightarrow{\Delta V_3}\rvert$, $\lvert\overrightarrow{\Delta V}\rvert_3$	magnitude of the third impulse vector

$\bar{x}$ denotes that the quantity x is a fixed scalar constant

$\vec{x}$ denotes that the quantity x is a vector in the inertial moon reference X, Y, Z coordinate system

β_1 generalized coast on the first transfer conic (between the first and second impulses)

β_2 generalized coast on the second transfer conic (between the second and third impulses)

μ_m fundamental gravitational parameter of the moon

Subscripts

0 denotes parking orbit

h denotes terminal hyperbola

METHOD

Both the two-impulse and three-impulse programs based on the accelerated gradient method required a specified fixed initial state vector from which the first impulse would occur. For this abort analysis a 2-hour delay time from SPS shutdown to the first abort maneuver (impulse) was assumed, thus establishing a fixed selenocentric state vector (SV_1, figs. 1 and 2) at the first abort point (ref. 1). This state vector was actually some point on a non-nominal, elliptical, lunar parking orbit.

Also needed in both programs was a criterion for specifying an acceptable class of lunar escape hyperbolas. The criterion used was to fix an asymptotic velocity vector and also a minimum allowable pericynthion. The associated numerical values for the fixed asymptotic velocity vector and a minimum allowable pericynthion were determined from the given selenocentric state vector at the moon's sphere of influence (SV_h) of a preassigned acceptable escape hyperbola (fig. 1). Determination of a fixed asymptotic velocity vector was deemed equivalent to the calculation of a velocity vector at infinity ($\vec{V}_\infty$) from the given state vector of this preassigned escape hyperbola. In mathematical notation, $\vec{R}_s$,

$\vec{V}_s \longrightarrow \vec{V}_\infty, \bar{r}_p$, where $\vec{R}_s$, $\vec{V}_s$ (or SV_h) was the given state vector and $\bar{r}_p$, the minimum allowable pericynthion value. By entering values of both $\bar{r}_p$ and $\vec{V}_\infty$ into the programs, an acceptable class of escape hyperbolas was generated (specified).

Furthermore, the given escape hyperbola must be in this class (that is, it must be an acceptable escape hyperbola). However, such a class of hyperbolas will have at least two-degrees-of-freedom since $\vec{V}_\infty, \bar{r}_p \longleftrightarrow (V_\infty)_x, (V_\infty)_y, (V_\infty)_z, e \geq \bar{e} \longleftrightarrow$ at least two-degrees-of-freedom (exactly six independent parameters must be specified, with μ_m fixed, in order to fix the escape hyperbola). Therefore, the programs will select the optimal acceptable escape hyperbola where optimal means minimum-ΔV.

Finally, in both programs, a period parameter, T1, was used in conjunction with the first transfer (after the first impulse) conic. Such a period parameter was used to insure that the first transfer conic be closed (that is, elliptical). In other words, using the period, the first transfer conic was not permitted to be parabolic or hyperbolic at the converged limit attained by the programs because it was anticipated that otherwise the second impulse might tend to occur at infinity.

The mathematical form of the two-impulse and three-impulse orbit transfer programs based on the accelerated gradient method was as follows:

$$\text{Set } F = \sum_{i=1}^{n} |\overrightarrow{\Delta V}|_i$$

where n = 2 and 3, respectively, for the two-impulse and three-impulse programs and $|\overrightarrow{\Delta V}|_i$ is the magnitude of the i^{th} impulse.

Minimize F subject to the following five non-linear constraints:

1. $a_1 - \bar{a}_1 = 0$
2. $\sin D_\infty - \overline{\sin D_\infty} = 0$
3. $a - \bar{a} = 0$
4. $RA - \overline{RA} = 0$
5. $\bar{r}_p - r_p \leq 0$

where (1) is the period constraint on the first transfer conic; (2) through (4) are constraints on $\vec{V}_\infty$ (spherical coordinates with respect to the celestial equator and the vernal equinox); and (5) is the minimum allowable pericynthion constraint.

With respect to the first constraint, the formula

$$T1 = 2\pi\sqrt{a_1^3/\mu_m}$$

was used to relate a_1, the semimajor axis of any first transfer ellipse, to the period, T1. When $T1 = \overline{T1}$ ($\overline{T1}$ being an input number to the program), $a_1 = \overline{a_1}$. The second constraint specifies the desired declination of $\vec{V}_\infty$, D_∞, with $\overline{\sin D_\infty}$ an input number. The third constraint specifies the desired magnitude of $\vec{V}_\infty$, $\bar{a}$ being the semimajor axis of the terminal escape hyperbola and an input number. The formula

$$\frac{1}{a} = \frac{2}{R} - \frac{V^2}{\mu_m}$$

reduces to $$\frac{1}{a} = \frac{-V_\infty}{\mu_m} \qquad \text{as } R \rightarrow \infty \text{ where } V_\infty = |\vec{V}_\infty|$$

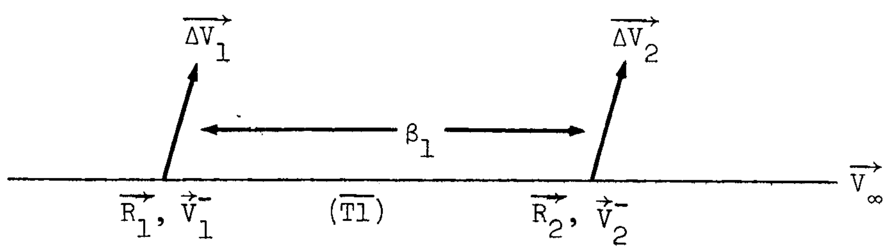
thus giving the relationship between V_∞ and a. The fourth constraint specifies the desired right ascension of $\vec{V}_\infty$, RA, with $\overline{RA}$ an input to the program; and the fifth constraint specifies the minimum allowable pericynthion, r_p, $\overline{r_p}$ being entered into the program.

The programs seek an optimal solution by iterations on the control parameters which are the inertial moon reference components of the $\vec{\Delta V}_i$ (ith impulse vector) and generalized (as given by Battin's theory) coast on each transfer conic. At the converged limit attained by the programs each constraint will be satisfied to the numerical accuracy of the computer (IBM 7094 - Double Precision).

All vector quantities such as $\vec{\Delta V}_i$ and $\vec{V}_\infty$ are in the inertial moon reference coordinate system (X, Y, Z) where the occupied focus for all conics obtained by the programs (including the elliptical parking orbit) is the moon's center. In both programs a fixed starting point (initial state vector) from which the first impulse would occur was specified (by $\vec{R}_1$, $\vec{V}_1^-$) on the fixed elliptical parking orbit. Also from the given selenocentric state vector at the moon's sphere of influence (SV_h) of a preassigned terminal escape hyperbola, the desired asymptotic velocity vector, $\vec{V}_\infty$, was computed and entered into the program by means of the fixed quantities $\overline{\sin D_\infty}$, $\bar{a}$, and $\overline{RA}$. Further, $\bar{r}_p$, the minimum allowable pericynthion of the optimal terminal hyperbola, was computed from this state vector and entered into the program. Since $\overline{\sin D_\infty} \neq 0$ and $\bar{r}_p$ was greater (numerically) than $(r_p)_0$, the pericynthion of the fixed elliptic parking orbit, the trajectories are best described as non-coplanar, minimum-ΔV, two-impulse (I-C-I) and three-impulse (I-C-I-C-I) orbit transfers from an inner ellipse to an outer $\vec{V}_\infty$ vector. The control parameters for both programs are the inertial moon reference components of the $\overrightarrow{\Delta V}_i$ (i^{th} impulse vector) and the generalized coast on each transfer conic, denoted by β_i (i_{th} transfer conic).

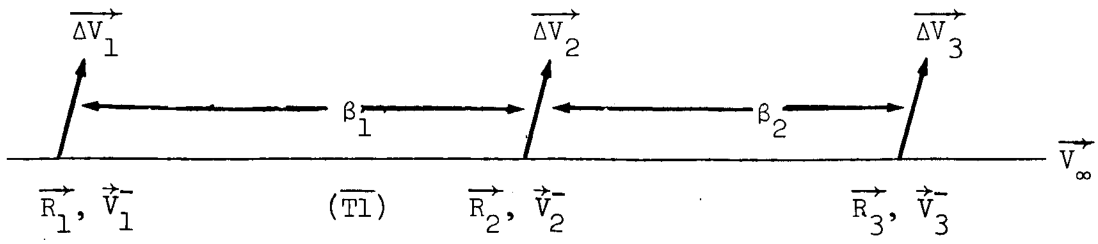
The following schematic sketches describe the two-impulse and three-impulse maneuvers:

(a) Two-impulse maneuver (I-C-I)



where $\overline{T1}$, the period of the first transfer conic, takes on a range of values, and $\vec{R}_i$, $\vec{V}_i^-$, ($i = 1, 2$) denotes the state vector immediately prior to the i^{th} impulse, $\vec{V}_i^-$.

(b) Three-impulse maneuver (I-C-I-C-I)



where $\overline{T1}$, the period of the first transfer conic, takes on a range of values and $\vec{R}_i, \vec{V}_i^-$, (i = 1, 2, 3) denotes the state vector immediately prior to the i^{th} impulse, $\overrightarrow{\Delta V}_i$.

Figure 1 describes the reference trajectory (ref. 1) used in this study. Figure 2 (ref. 2) describes a typical minimum-ΔV, three-impulse transfer sequence where conic #0 is the elliptical parking orbit in the programs used in this document.

Table I presents a summary of solutions obtained in both the two-impulse and three-impulse programs over the period range T1 = 8, 20, 30, 40, 50, 60, and 70 hours. Because the numerical value of F seemed to taper off for $T1 = \overline{T1} = 40$ and 50 hours, the programs were run with the period (or first) constraint removed. It was thought that by doing this the two-impulse and three-impulse programs would have an extra-degree-of-freedom to optimize the period T1; that is, both programs would have the freedom to select the exact value of T1 which would give the smallest possible numerical value of F (the sum of the magnitudes of the impulses). This implies that the programs would have the freedom to choose the optimal first transfer conic while minimizing F subject to getting on a specified $\vec{V}_\infty$ (with minimum allowable hyperbolic pericynthion) by means of the two- or three-impulses.

The optimal value of T1 selected could conceivably have been very large, $T1 \to \infty$ (a parabola). However, when the modified programs were actually run, the optimal values of T1 turned out to be finite. In particular, the optimal values were T1" = 46.48798 (finite, two-impulse case) and T1' = 46.48817 (finite, three-impulse case). Furthermore, $T1'' \simeq T1_0$, $T1' \simeq T1_0$, and $T1_0 = 46.48918$. The deviation from an ideal maneuver ($T1 = \infty$) was more pronounced than previously predicted.

The numerical closeness of T1" and T1' to $T1_0$, the period of the parking conic, appeared to be more than a coincidence. Therefore, as a test case, the period constraint (for the first transfer conic) was put back into both programs with $T1 = \overline{T1} = T1_0$. Comparing the results obtained from the modified two- and three-impulse programs (in which the period constraint was deleted) and the respective programs with the period constraint contained such that $T1 = T1_0$, it was noted that the optimal trajectories (maneuvers) and numerical values of F obtained were the same, within computer accuracy. It was concluded that $T1'' = T1_0$ for the two-impulse case and $T1' = T1_0$ for the three-impulse case, and more importantly, the trajectories and values or F obtained were the same in each case.

Furthermore, it appeared that the optimal trajectories (maneuvers) generated in the $T1 = T1_0$ cases (two-impulse and three-impulse) were of a degenerate type in that the first impulse ($\overrightarrow{\Delta V}_1$) tended to vanish. As a test case for the three-impulse, $T1 = T1_0$, degenerate-type trajectory, a two-impulse program with an allowable initial coast (C-I-C-I; starting point at $\vec{R}_1$, $\vec{V}_1$, with no period constraint) was implemented, using as an initial guess the three-impulse, $T1 = T1_0$, degenerate-type maneuver. The optimal trajectory achieved by the two-impulse, C-I-C-I program was the same, within computer accuracy, as the optimal, three-impulse, $T1 = T1_0$, degenerate-type maneuver. Therefore, it may be concluded that the optimal, three-impulse, $T1 = T1_0$, degenerate-type maneuver is a C-I-C-I, two-impulse trajectory ($\overrightarrow{\Delta V}_1 = 0$). Similarly, it follows that the optimal, two-impulse, $T1 = T1_0$, degenerate-type maneuver is a C-I, one-impulse trajectory ($\overrightarrow{\Delta V}_1 = 0$) by comparing the order of magnitudes of $|\overrightarrow{\Delta V}_1|$ in the degenerate two-impulse case and degenerate three-impulse case.

Results obtained with the period constraint removed and $T1 = T1_0$ are included in table I. Table II shows the relationship of F to T1 for all values of T1 attempted thus far in the two-impulse (I-C-I) and three-impulse (I-C-I-C-I) programs. Figure 3 shows velocity expenditure as a function of T1.

The programs could have chosen a hyperbolic first transfer conic. Furthermore, under some sets of initial conditions (specified $\vec{V}_\infty$, minimum pericynthion, etc.), the impulse programs could choose optimal parabolic or hyperbolic first transfer conics. If the first transfer conic selected was parabolic or hyperbolic, it might or might not force the second impulse to occur at ∞.

The initial data for the two-impulse (I-C-I, 7-parameter) and three-impulse (I-C-I-C-I, 11-parameter) programs are shown in table III.

DISCUSSION OF SOLUTIONS AND CONCLUSIONS

The three-impulse maneuvers led to the following percentage decreases in F (the sum of the magnitudes of the impulses) from two-impulse maneuvers with the same period. For T1 = 8 hours, there was a 30 percent decrease in F; for T1 = 20 hours, a 3 percent decrease; and for T1 = 30 hours, a 1.25 percent decrease. For T1 = 40 hours, $T1_0$ hours, T1" hours and T1' hours (obtained by deleting g_1 in the two-impulse and three-impulse programs, respectively), 50 hours, 60 hours, and 70 hours there was effectively no decrease in F (decreases of the order of 0.1 percent). However, the optimal three-impulse maneuvers (solutions) for periods of 8 hours, 20 hours, and 30 hours are not acceptable because $(r_p)_1$, the pericynthion altitude of the first transfer conic, had a numerical value smaller than the mean lunar radius (938.49256551 n. mi.), in each case. Footnote notation, b, in table I designates the unacceptable values of $(r_p)_1$. The two-impulse maneuver for T1 = 8 hours is not acceptable for the same reason. Therefore, for T1 = 8 hours it would be necessary to input an additional constraint on the pericynthion $(r_p)_1$ in the three-impulse program. By doing this, it is likely that the value of F would increase to approximately that attained in the two-impulse, 8-hours case (itself unacceptable). Similarly placing an additional constraint on $(r_p)_1$ in the three-impulse, T1 = 20 hour- and T1 = 30 hour-cases would probably increase the values of F to approximately those attained by the two-impulse programs for the corresponding periods (themselves acceptable). Since there was no effective decrease in F in the optimal three-impulse maneuvers from the optimal two-impulse maneuvers for periods of T1 = 40 hours and larger, and noting (from the tables and fig. 3) that the value of F is minimal at T1 = T1" = $T1_0$ in the two-impulse program and at T1 = T1' = $T1_0$ in the three-impulse program, it can be concluded that the best minimum F solution is either the degenerate, two-impulse, T1 = T1" = $T1_0$ maneuver or the degenerate, three-impulse, T1 = T1' = $T1_0$ maneuver. The second impulse $(\overrightarrow{\Delta V_2})$ becomes very small for the degenerate, three-impulse, T1 = T1' = $T1_0$

maneuver and for three-impulse maneuvers of T1 = 40 hours and larger. As a result it was concluded that the best solutions to the problem of determining a class of minimum-ΔV, two-impulse and three-impulse abort trajectories onto an optimal lunar escape hyperbola partially specified by a fixed asymptotic velocity vector and minimum pericynthion are the degenerate two-impulse, $T1 = T1'' = T1_0$ maneuver, and the degenerate three-impulse, $T1 = T1' = T1_0$ maneuver, and possibly the two-impulse, T1 = 40 maneuver (see tables I and II). The two-impulse, T1 = 40 maneuver, however, has a value of $(r_p)_1$ barely larger than the lunar radius (approximately 20 n. mi.) and thus may or may not be acceptable.

CONCLUDING REMARKS

When the first transfer conic was permitted to be either parabolic or hyperbolic (by deleting the period constraint in the programs), the second impulse did not occur at ∞, but at some point reached after a finite coast along this first transfer conic. Therefore, by deleting the period parameter, it might yet be possible to obtain a minimum-ΔV solution where the program has the freedom to select the optimal first transfer conic whether it be an ellipse, parabola, or hyperbola. Further, if the input period parameter is allowed to become arbitrarily large, using the formula $T1 = 2\pi\sqrt{\frac{a^3}{\mu}}$, it can be seen as $T1 \to \infty$, $a \to \infty$. This case ($T1 = \infty$) would correspond to forcing the first transfer conic to be a parabola. Of course, entering an arbitrarily large period into the program would lead to numerical problems (of the first order). Perhaps entering $\bar{e} = 1$ (where $\bar{e}$ denotes the eccentricity of the first transfer conic) in place of $T1 = \infty$, by means of the constraint $e - \bar{e} = 0$, would accomplish the same purpose. It might be possible to obtain a minimum-ΔV solution for this case. Also, the unconstrained second transfer conic, considered in the three-impulse orbit transfer program, could be parabolic, hyperbolic, or elliptic. Again, given the freedom to select the optimal second transfer conic by means of the second impulse, the program may be able to obtain a minimum-ΔV solution (second impulse does not occur at ∞) or the program may not attain a converged limit due to the second impulse tending to ∞. In any case, the importance of the period parameter (or period constraint) in impulsive orbit transfer programs is deserving of further study.

TABLE I.- SUMMARY OF SOLUTIONS FOR TWO-IMPULSE AND THREE-IMPULSE PROGRAMS

(a) Three-impulse (I-C-I-C-I)

Solution	1	2	3	4	[a]5	[a]6	7	8	9
T1, hr	8	20	30	40	46.48817	46.48918	50	60	70
F, fps	4014.324385	1914.148691	1 574.078653	1 443.794598	1 397.173158	1 397.173680	1 424.604401	1 486.010919	1 530.381929
$\|\Delta \vec{V}_1\|$, fps	1841.170240	464.641557	197.610292	64.978042	.064236	.063609	28.941267	94.634068	143.923384
$\Delta \vec{V}_1$									
Δu_1, fps	-1642.174440	-423.604030	-192.830348	-62.086135	-.012652	1.309095	23.186430	-14.902725	112.619980
Δv_1, fps	-816.174410	-188.877543	-40.387327	.507145	.015683	.017760	15.681615	53.695206	83.845002
Δw_1, fps	-164.408530	-27.868863	15.334387	19.162383	.018478	.018149	7.353228	21.494427	31.633728
a_1, n. mi.	2531.132247	4662.385578	6 109.453459	7 401.075609	8 181.189935	8 181.309095	8 588.187462	9 698.152448	10 747.815534
e_1, n. mi.	.847910	.822379	.854959	0.870506	0.875476	0.875477	0.881048	0.89413246	0.904291
$(r_p)_1$, n. mi.	[b]384.958763	[b]828.138498	[b]886.120013	958.398437	1 018.753817	1 018.758776	1 021.580597	1 026.719532	1 028.664803
$(r_a)_1$, n. mi.	4677.305730	8496.632635	11 332.7869	13 843.752781	15 343.626110	15 343.859410	16 154.794310	18 369.585353	20 466.966241
$\vec{R}_2$									
x_2, n. mi.	3596.348654	7281.335893	6 888.055897	5 968.900330	6 517.738673	6 517.708711	6 119.686074	6 076.072100	6 741.191416
y_2, n. mi.	2806.755039	-28.000642	-1 763.906465	-2 770.686103	-3 262.384629	-3 262.446034	-3 396.878983	-3 691.738727	3 980.540798
z_2, n. mi.	977.124718	-1365.153921	-2 310.717605	-2 711.256070	-3 133.034065	-3 133.199270	-3 126.292358	-3 283.872604	-3 578.410926
$\vec{V}_2^-$									
u_2^-, fps	721.105855	-961.878207	-1 666.732128	-2 012.006398	-1 892.528102	-1 892.536291	-1 985.508084	-2 018.645569	-1 926.472265
v_2^-, fps	-506.483855	-764.081124	-119.983952	-87.753966	-16.776349	-16.757151	71.710891	182.080911	192.878524
w_2^-, fps	-425.465809	-268.455675	63.791862	318.330308	344.350058	344.362904	411.242624	480.453109	470.177037
t_2, hr	9.036993	12.154230	23.159177	33.83824	39.62198	39.62300	43.47027	53.47725	62.88913
$\|\Delta \vec{V}_2\|$, fps	625.045401	144.201987	81.366429	7.691941	14.064440	14.065139	16.917821	32.857936	49.008385
$\Delta \vec{V}_2$									
Δu_2, fps	406.274581	-45.592485	-52.122987	-5.689085	-10.556260	-10.556912	-13.512488	-27.998415	-41.961642
Δv_2, fps	-377.500454	-123.353581	-59.131251	-5.125391	-9.132924	-9.133247	-10.099986	-17.194359	-25.202564
Δw_2, fps	-288.298625	-59.155971	-20.178754	.728430	1.720507	1.720514	1.269748	-.288662	-2.422751
a_2, n. mi.	2951.784214	4927.448959	6 422.689206	7 448.113887	8 230.193382	8 230.321152	8 729.044346	10 066.732552	11 401.831264
e_2, n. mi.	.588354	0.767462	.836420	0.869537	0.873600	0.873602	0.879191	0.891375	0.900073
$(r_p)_2$, n. mi.	1215.091299	1145.820674	1 050.62064	971.702899	1 046.613380	1 046.619923	1 054.544035	1 093.497852	1 139.351657
$(r_a)_2$, n. mi.	4688.477142	8709.077216	11 794.7578	13 924.524865	15 513.773385	15 514.022380	16 403.544629	19 039.967186	2 166.431082

[a]For solution 5, T1 = T1'; for solution 6, T1 = $T1_0$. Solutions 5 and 6 should be considered identical because of computer accuracy. (Hence T1' = $T1_0$.) For the same reason, the $\Delta \vec{V}_1$ for both solutions should be considered a zero impulse.

[b]Designates an inacceptable value of $(r_p)_1$.

TABLE I.- SUMMARY OF SOLUTIONS FOR THE TWO-IMPULSE AND THREE-IMPULSE PROGRAMS - Continued

(a) Three-impulse (I-C-I-C-I) - Concluded

Solution	1	2	3	4	[a]5	[a]6	7	8	9
T1, hr	3	20	30	40	46.48817	46.48918	50	60	70
$\vec{R}_3$									
x_3, n. mi.	-1 101.326842	-645.668634	-319.645848	-39.211109	-111.458144	-111.462142	-103.776140	-124.525985	-171.099439
y_3, n. mi.	-518.800148	-1 056.038978	-1 311.290696	-1 562.815906	-1 697.711547	-1 697.727114	-1 732.847281	-1 805.564285	-1 854.086933
z_3, n. mi.	-102.179757	-492.341921	-700.539244	-899.714769	-965.625941	-965.634241	-986.928771	-1 024.438442	[illegible]
$\vec{V}_3^-$									
u_3^-, fps	-2 802.768770	-3 879.971411	-4 063.184815	-3 863.030039	-3 574.625439	-3 574.598985	-3 535.738278	-3 438.775323	-3 358.098201
v_3^-, fps	4 439.481200	3 705.490769	3 141.425926	2 743.705734	2 770.369679	2 770.369398	2 760.722736	2 769.562760	2 798.896071
w_3^-, fps	3 070.978427	2 866.068483	2 581.213791	2 306.339392	2 288.569423	2 288.564686	2 269.835093	2 253.268940	2 256.059899
t_3, hr	24.621700	17.790702	27.630277	37.47061	43.89688	43.89789	47.40402	57.40355	67.41275
$\lvert\Delta\vec{V}_3\rvert$, fps	1 548.108744	1 305.30510	1 295.101935	1 371.124568	1 383.044502	1 383.044904	1 378.745321	1 358.518907	1 337.450167
$\Delta\vec{V}_3$									
Δu_3, fps	-673.010074	-988.662582	-1 123.842146	-1 257.729378	-1 248.893649	-1 248.892806	-1 246.132677	-1 222.965409	-1 194.296188
Δv_3, fps	1 152.289455	646.159101	442.937895	319.587016	364.298960	364.233031	364.448950	376.215938	392.232169
Δw_3, fps	784.810092	555.739226	466.983871	442.677667	469.481832	469.482924	463.970783	456.498359	456.709308
e_h	1.494567	1.494567	1.494567	1.513421	1.558455	1.558459	1.565790	1.586680	1.605939
g_1, e.r.	-.11102230E-15	-.11102230E-15	-.11102230E-15	-0.17763568E-14	--	-0.11102230E-14	0.	0.15543122E-14	-0.48849813E-14
g_2	-.97144515E-16	-.18735014E-15	-.19290125E-14	-0.13415658E-12	-0.35832448E-13	-0.59702243E-13	0.25812685E-14	0.29282132E-14	-0.46629367E-14
g_3, e.r.	.39412918E-14	-.15543122E-14	-.17652546E-13	-0.73208107E-12	-0.11035617E-12	-0.31152858E-12	0.15765167E-13	0.14543922E-13	-0.99920073E-14
g_4, rad	-.33306691E-15	.61062267E-14	.13988810E-13	0.90316643E-12	0.22137347E-12	0.35393910E-12	-0.24757973E-13	-0.13103029E-13	0.17874591E-13
g_5, e.r.	.13877788E-15	-.94368957E-15	-.45241588E-14	-0.13485821E-01	-0.45698973E-01	-0.45702170E-01	-0.50946015E-01	-0.65888952E-01	-0.79664754E-01

[a]For solution 5, $T1 = T1'$; for solution 6, $T1 = T1_0$. Solutions 5 and 6 should be considered identical because of computer accuracy. (Hence $T1' = T1_0$.) For the same reason, the $\Delta\vec{V}_1$ for both solutions should be considered a zero impulse.

TABLE I.- SUMMARY OF SOLUTIONS FOR TWO-IMPULSE AND THREE-IMPULSE PROGRAMS - Concluded

(b) Two-impulse (I-C-I)

Solution	1	2	3	4	a5	a6	7	8	9
T1, hr	8	20	30	40	46.48798	46.48918	50	60	70
F, fps	5678.118571	1972.335078	1 594.470658	1 444.04023	1 400.074149	1 400.074792	1 425.80428	1 487.045411	1 532.301740
$\lvert\Delta\vec{V}_1\rvert$, fps	3060.690476	549.31446	206.498721	65.581917	.075981	.074937	29.981746	94.760961	142.982615
$\Delta\vec{V}_1$									
Δu_1, fps	-2955.293386	-388.584156	-174.034137	-61.440792	-.010746	-.001371	23.945333	76.360575	114.848295
Δv_1, fps	325.470386	-367.724153	-110.095556	-1.543109	.005294	.007867	13.3184	48.985701	76.601169
Δw_1, fps	726.729522	-124.610064	-15.257959	22.883033	.047443	.046616	12.171398	27.369695	37.231096
a_1, n. mi.	2531.132282	4662.385578	6 109.453459	7 401.07561	8 181.167205	8 181.309095	8 588.18746	9 698.152792	10 747.815534
e_1	0.883006	.775749	.841658	.870520	0.875477	0.875478	.881098	0.894006	0.903937
$(r_p)_1$, n. mi.	b 296.128038	1045.542420	967.382932	958.291326	1 018.746205	1 018.752267	1 021.15104	1 027.948706	1 032.463462
$(r_a)_1$, n. mi.	4766.136514	8279.228719	11 251.52396	13 843.85995	15 343.588226	15 343.865873	16 155.2239	18 368.356862	20 463.167641
$\vec{R}_2$									
x_2, n. mi.	2181.579463	-366.3452	-95.174313	-10.67441	-29.323073	-29.323040	-32.100544	4.059862	37.989997
y_2, n. mi.	1132.702283	-1278.353156	-1 456.490414	-1 569.439313	-1 728.208957	-1 728.229896	-1 752.67817	-1 842.640021	-1 916.413659
z_2, n. mi.	271.578873	-673.170923	-825.272319	-909.109647	-1 007.995691	-1 008.008434	-1 014.7184	-1 070.262286	-1 118.500637
$\vec{V}_2^-$									
u_2^-, fps	1953.232956	-3962.683233	-4 001.7351	-3 872.34853	-3 590.625613	-3 590.593860	-3 553.6588	-3 459.749164	-3 389.864434
v_2^-, fps	2583.060260	3162.264685	2 801.79931	2 717.06443	2 666.283065	2 666.277847	2 695.03767	2 666.256972	2 636.193737
w_2^-, fps	1139.828310	2534.075426	2 336.7636	2 278.904134	2 235.532410	2 235.525144	2 219.69795	2 174.291082	2 141.938758
t_2, hr	6.284851	17.32581	27.442035	37.46268	43.86772	43.86892	47.38282	57.36438	67.34597
$\lvert\Delta\vec{V}_2\rvert$, fps	2617.428095	1423.02055	1 387.971925	1 378.458297	1 399.998198	1 399.999884	1 395.8225	1 392.284433	1 389.319132
$\Delta\vec{V}_2$									
Δu_2, fps	355.871230	-1205.572024	-1 254.02327	-1 266.584175	-1 247.222165	-1 247.221523	-1 263.6613	-1 271.292616	-1 272.527141
Δv_2, fps	2269.115766	541.971896	379.850363	303.891124	309.199105	309.200827	325.901954	330.765288	327.934543
Δw_2, fps	1255.149011	527.114781	457.826727	451.178468	555.722611	555.727276	495.39412	461.373218	450.933679
e_h	1.494567	1.494567	1.494567	1.510830	1.553824	1.553829	1.559633	1.575446	1.587668
g_1, e.r.	.55511151E-16	.11102230E-15	-.11102230E-15	-.44408921E-15	---	0.66613382E-15	0.	0.66613382E-15	0.24424907E-14
g_2	.44408921E-15	.41633363E-16	.69388939E-15	.90205621E-15	-0.12490009E-15	-0.55511151E-16	-.11102230E-14	0.88817842E-15	0.38857806E-15
g_3, e.r.	.45519144E-14	.35305092E-13	.42743586E-13	.42299497E-13	0.14155344E-13	-0.65503159E-14	-.39690473E-13	0.69833028E-13	0.54845017E-13
g_4, rad	-.77715612E-15	-.18873791E-14	-.16653345E-14	-.19984015E-14	0.18873791E-14	-0.55511151E-15	.16653345E-14	0.48849813E-14	0.45519144E-14
g_5, e.r.	.21094238E-14	.19151347E-14	.28310687E-14	-.11632568E-01	-0.42386551E-01	-0.42390225E-01	-.46541853E-01	-0.57853302E-01	-0.66595268E-01

[a]For solution 5, T1 = T1"; for solution 6, T1 = $T1_0$. Solutions 5 and 6 should be considered identical because of computer accuracy. (Hence, T1" = $T1_0$.) For the same reason, the $\Delta\vec{V}_1$ for both solutions should be considered a zero impulse.

[b]Designates an unacceptable value of $(r_p)_1$.

TABLE II.- VELOCITY EXPENDITURE FOR VARIOUS PERIODS

OF THE FIRST TRANSFER CONICS

(a) Three-impulse (I-C-I-C-I)[a]

Period of the 1st transfer conic, T1, hr	Velocity expenditure, F, fps
8	4014.32438505
20	1914.48691
30	1574.078653
40	1443.79459792
[b]46.48817 (T1')	1397.17315781
[c]46.48918 ($T1_0$)	1397.17367967
50	1424.60440120
60	1486.01091917
70	1530.38192903
(b) Two-impulse (I-C-I)[a]	
8	5678.11857141
20	1972.335078
30	1594.470658
40	1444.04023
[b]46.48798 (T1")	1400.07414947
[c]46.48918 ($T1_0$)	1400.07479176
50	1425.80428
60	1487.04541056
70	1532.30173953

[a]Indicates impulse sequence; I = impulse, C = coast.

[b]Resulting from deleting first constraint.

[c]Period of parking orbit.

TABLE III.- INITIAL DATA FOR TWO-IMPULSE[a] AND THREE-IMPULSE[b] PROGRAMS

(a) Fixed elliptic lunar parking orbit

Fixed initial state vector (SV_1):	
x_1,	
e.r.	0.86070365
n. mi.	2964.2062
y_1,	
e.r.	0.85888027
n. mi.	2957.9266
z_1,	
e.r.	0.34261064
n. mi.	1179.9283
$\bar{u}_1 = \dot{\bar{x}}_1$,	
e.r./hr	0.51275146
fps	2980.4730
$\bar{v}_1 = \dot{\bar{y}}_1$,	
e.r./hr	0.14697792
fps	854.3393
$\bar{w}_1 = \dot{\bar{z}}_1$,	
e.r./hr	-0.00976763
fps	-56.776342
$\bar{a}_0$,	
e.r.	2.237557111
n. mi.	8181.30912958
$\bar{e}_0$,	0.87547398
$\dot{T1}_0$, hr	46.48918442

[a] 7-parameter.

[b] 11-parameter.

TABLE III.- INITIAL DATA FOR TWO-IMPULSE[a] AND THREE IMPULSE[b] PROGRAMS - Continued

(a) Fixed elliptic lunar parking orbit - Concluded

$(\bar{r}_p)_0$,	
e.r.	0.29582042
n. mi.	1018.78587956
$(\bar{r}_a)_0$,	
e.r.	4.45532180
n. mi.	15 343.83237900
$\bar{r}_{moon}$,	
e.r.	0.272506
n. mi.	938.49256551
R_1,	
e.r.	1.26327675
n. mi.	4350.64122647
V_1^-,	
e.r./hr	0.53349036
fps	3101.02207217

[a]7-parameter.

[b]11-parameter.

TABLE III.- INITIAL DATA FOR TWO-IMPULSE[a] AND THREE-IMPULSE[b] PROGRAMS - Concluded

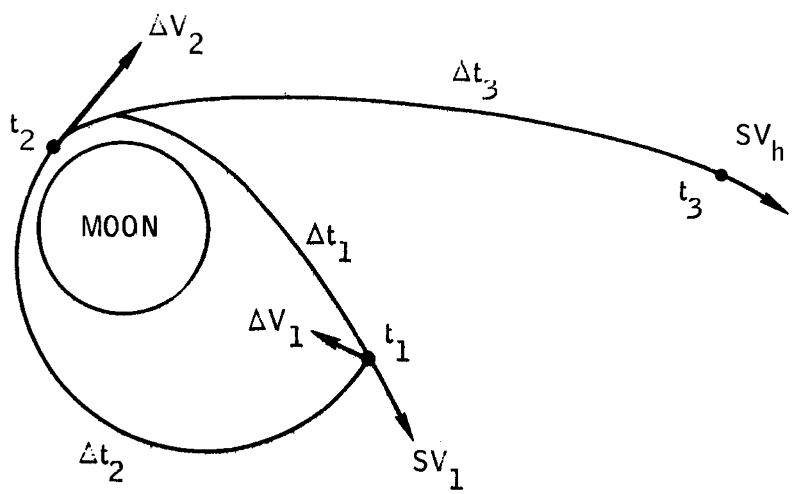
(b) Class of terminal escape hyperbolas[c]

$\bar{a} = \overline{a_h}$,	
e.r.	-0.71530711
n. mi.	-2463.47017971
$\lvert\vec{V}_\infty\rvert = \bar{V}_\infty$,	
e.r./hr.	0.58510488
fps.	3401.04207889
$\overline{\sin D_\infty}$	0.46038510
$\overline{RA}$, rad.	1.3303722
$\overline{r_p} = (\overline{r_p})_h$,	
e.r.	0.35376753
n. mi.	1218.35187784

[a]7-parameter.

[b]11-parameter.

[c]See constraints 2-5.



$\Delta V_1 = \;\; 800.00$ fps

$\Delta V_2 = 1452.26$ fps

Total $\Delta V = 2252.26$ fps

$t_1 = 23.93432$ hr

$t_2 = 37.55932$ hr

$t_3 = 50.764203$ hr

Initial orbit:

$a = 8181.3093$ n. mi.
$e = 0.87547398$
$i = 148.38377°$
at t_1

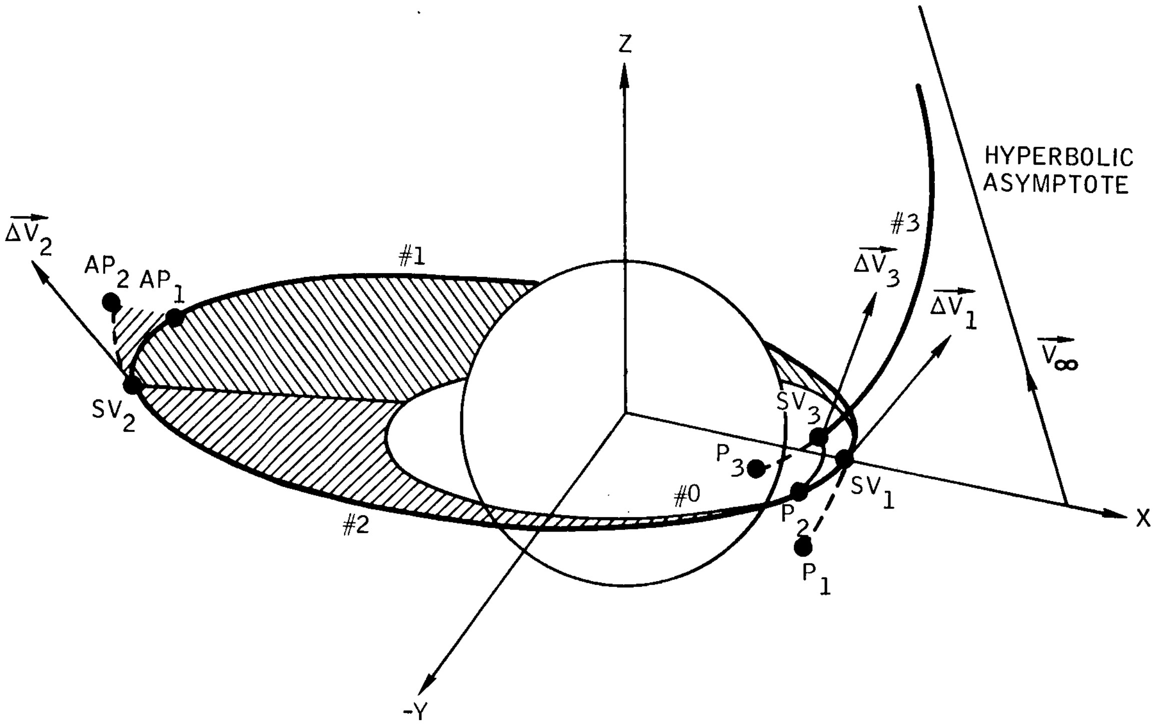
Intermediate orbit:

$a = 4071.0751$ n. mi.
$e = 0.73264273$
$i = 148.59948°$
at t_2

SV_1 = selenocentric state vector at first abort point

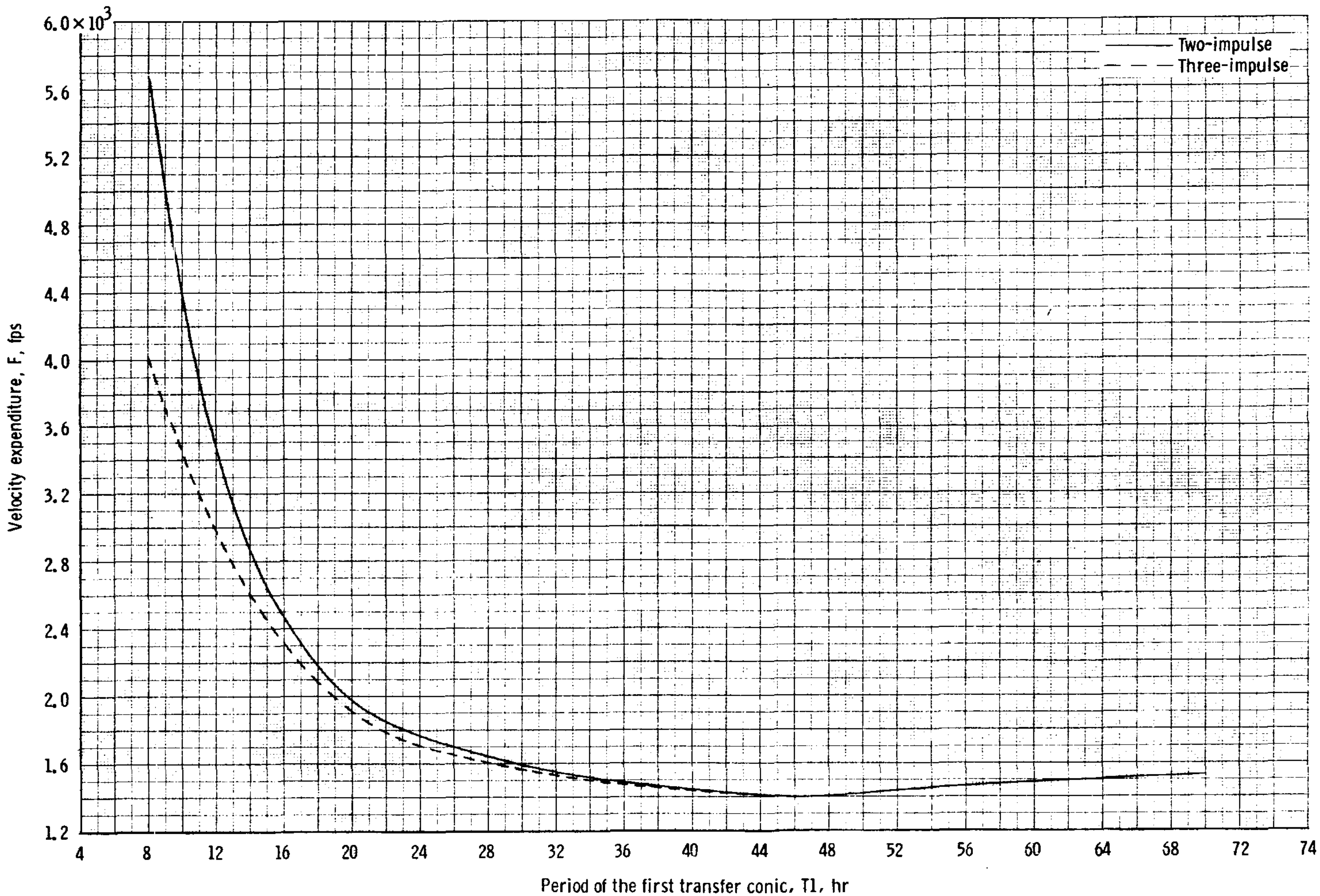
SV_h = selenocentric state vector at LSOI

Figure 1.- Reference trajectory.



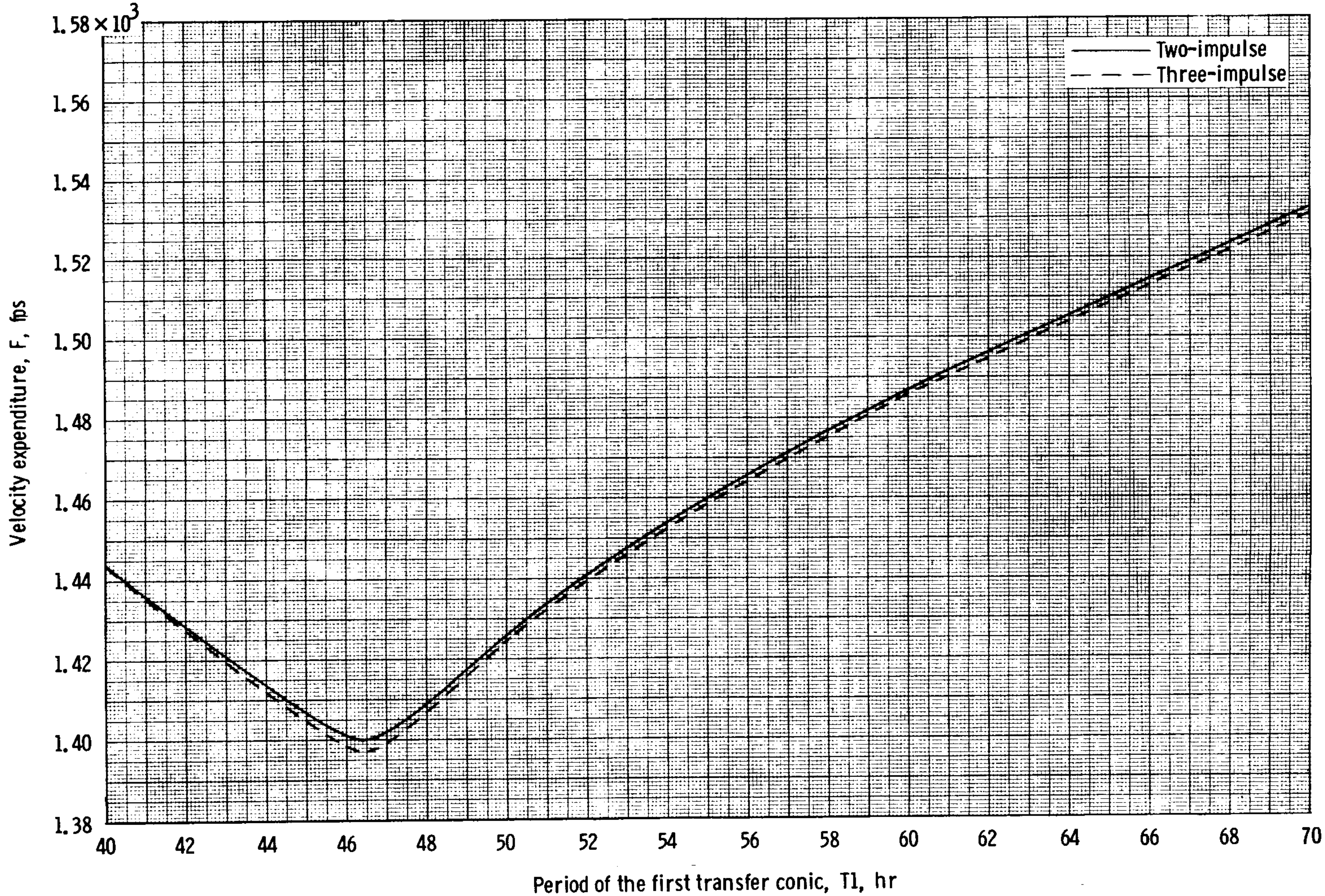
#0	ELLIPTICAL LUNAR PARKING ORBIT
#1	1ST TRANSFER ELLIPSE
#2	2ND TRANSFER ELLIPSE
#3	TERMINAL HYPERBOLA
SV_1, SV_2, SV_3	POINTS OF APPLICATION OF 1ST, 2ND, AND 3RD IMPULSES
$\overrightarrow{\Delta V_1}$, $\overrightarrow{\Delta V_2}$, $\overrightarrow{\Delta V_3}$	1ST, 2ND, AND 3RD IMPULSES
AP_1, AP_2	APOCYNTHION POINTS OF 1ST AND 2ND TRANSFER CONICS
P_1, P_2, P_3	PERICYNTHION POINTS OF 1ST TRANSFER CONIC, 2ND 3FER TRANSFER CONIC, AND TERMINAL HYPERBOLA

Figure 2.- Geometry for a typical minimum ΔV, three-impulse transfer sequence.



(a) 4 hours to 70 hours.

Figure 3. - Velocity expenditure as a function of the period of the first transfer conic.



(b) 40 hours to 70 hours.

Figure 3. - Concluded.

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